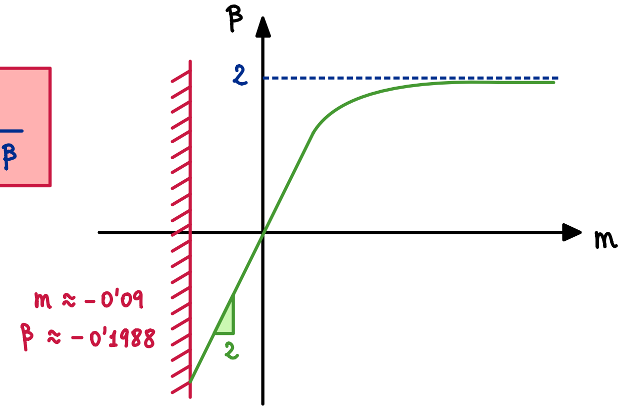
Capa Límite de Falkner-Skan

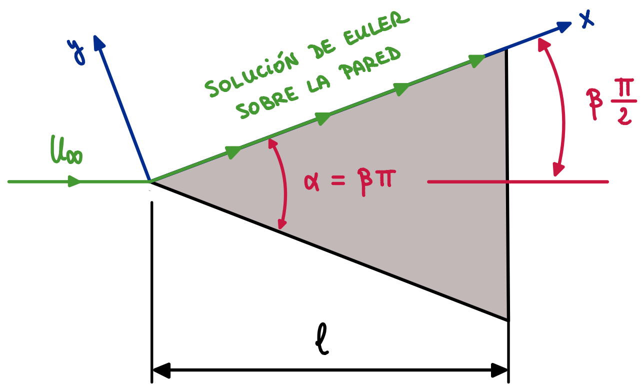
Podemos encontrar soluciones autosemejantes de las ecuaciones de CL con $\frac{du_e}{dx} \neq 0$ (CON GRADIENTE DE PRESIONES), y son relevantes.

$\boxed{U_e = A x^m}$

$\boxed{\beta = \frac{2m}{1+m} \rightleftarrows m = \frac{\beta}{2-\beta}}$

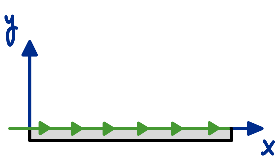
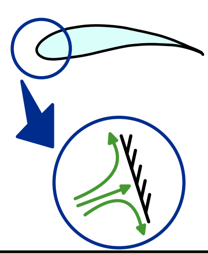
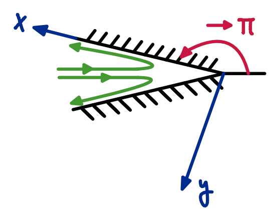
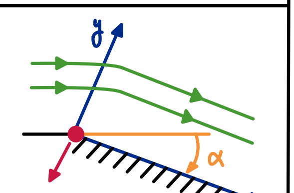


Obtención de A (suponiendo $0 \leq m \leq 1$):



$$\frac{U_e}{U_\infty} = g(\beta)\left(\frac{x}{\ell}\right)^m \longrightarrow U_e = U_\infty \underbrace{\frac{g(\beta)}{\ell^m}}_{A} x^m \longrightarrow$$

$$\longrightarrow \boxed{A = U_\infty \frac{g(\beta)}{\ell^m}}$$

	I	II	III	IV
β	0	1	$\rightarrow 2$	$-0'1988 < \beta < 0$
m	0	1	$\rightarrow \infty$	$-0'09 < m < 0$
α	0	$\frac{\pi}{2}$	$\rightarrow \pi$	$-17^\circ < \alpha < 0^\circ$
ESQU	 PLACA PLANA			 SINGULARIDAD EN $x = 0$

$U_e = 2U_\infty \text{ sen } \theta \longrightarrow U_e \approx 2U_\infty \theta = \frac{2U_\infty}{R} x^1 \longrightarrow$

$\boxed{A_{cilindro} = \frac{2U_\infty}{R}}$ $\boxed{m_{cilindro} = 1}$

NOTA

$$u_e \frac{du_e}{dx} = Ax^m \cdot Amx^{m-1} = mA^2 x^{2m-1} = \underbrace{(Ax^m)^2}_{u_e^2} m x^{-1} \longrightarrow \boxed{u_e \frac{du_e}{dx} = \frac{m u_e^2}{x} = \frac{\beta}{2-\beta}\frac{u_e^2}{x}}$$

El problema a resolver es:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad y = 0:\ u = 0;\ v = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\beta}{2-\beta}\frac{u_e^2}{x} + \nu \frac{\partial^2 u}{\partial y^2} \qquad y \to \infty:\ u \to u_e(x) = Ax^m$$

$$x = 0:\ u = u_e\big|_{x=0}$$

EXISTE SOLUCIÓN DE SEMEJANZA

Introducimos la función de corriente ψ:

$$\boxed{\psi = \sqrt{(2-\beta)\,\nu\, x\, u_e(x)}\; f(\eta)} \qquad \boxed{\eta = y\sqrt{\frac{u_e(x)}{(2-\beta)\,\nu\, x}}}$$

→ Simplificará la EDO que obtendremos

Derivadas de la variable de semejanza respecto a x, y:

$$\frac{\partial \eta}{\partial x} = \frac{\partial}{\partial x}\left\{ y\sqrt{\frac{u_e(x)}{(2-\beta)\nu x}}\right\} = \frac{\partial}{\partial x}\left\{ y\sqrt{\frac{Ax^{m-1}}{(2-\beta)\nu x}}\right\} = \sqrt{\frac{A}{(2-\beta)\nu}}\; y\,\frac{m-1}{2}\, x^{\frac{m-1}{2}-1} =$$

$$= \frac{m-1}{2}\underbrace{y\sqrt{\frac{Ax^m}{(2-\beta)\nu x}}}_{\eta}\;\underbrace{x^{\frac{m-1}{2}-1-\frac{m-1}{2}}}_{x^{-1}} = \frac{m-1}{2}\frac{\eta}{x} \longrightarrow \boxed{\frac{\partial \eta}{\partial x} = \frac{\beta-1}{2-\beta}\frac{\eta}{x}}$$

$$\frac{\partial \eta}{\partial y} = \frac{\partial}{\partial y}\left\{ y\sqrt{\frac{u_e(x)}{(2-\beta)\nu x}}\right\} = \sqrt{\frac{u_e(x)}{(2-\beta)\nu x}} = \frac{\eta}{y} \longrightarrow \boxed{\frac{\partial \eta}{\partial y} = \sqrt{\frac{u_e(x)}{(2-\beta)\nu x}} = \frac{\eta}{y}}$$

$\boxed{u}$

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}\left[\sqrt{(2-\beta)\nu x\, U_e(x)}\; f(\eta)\right] = \sqrt{(2-\beta)\nu x\, U_e(x)}\; \underbrace{\frac{df}{d\eta}}_{f'}\, \frac{\partial \eta}{\partial y} =$$

$$= \sqrt{(2-\beta)\nu x\, U_e(x)}\; f' \sqrt{\frac{U_e(x)}{(2-\beta)\nu x}} = U_e(x)\, f' \longrightarrow \boxed{u = U_e(x)\, f'}$$

$\boxed{v}$

$$v = -\frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x}\left[\sqrt{(2-\beta)\nu x\, U_e(x)}\; f(\eta)\right] = -\sqrt{(2-\beta)\nu}\left[\frac{U_e(x) + \underbrace{\boxed{x\,\frac{dU_e(x)}{dx}}}_{\frac{\beta}{2-\beta}U_e(x)}}{2\sqrt{x\, U_e(x)}}\, f \; + \right.$$

$$\left. + \sqrt{x\, U_e(x)}\; \underbrace{\frac{df}{d\eta}}_{f'}\, \frac{\partial \eta}{\partial y}\right] = -\sqrt{(2-\beta)\nu}\left[\sqrt{\frac{U_e(x)}{x}}\, f + \frac{\beta - 1}{2-\beta}\,\frac{\eta}{x}\, f' \sqrt{\frac{U_e(x)}{x}}\right] =$$

$$= \sqrt{\frac{\nu\, U_e(x)}{(2-\beta)\, x}}\left[-f + (1-\beta)\,\eta f'\right] \longrightarrow \boxed{v = \sqrt{\frac{\nu\, U_e(x)}{(2-\beta)\, x}}\left[(1-\beta)\,\eta f' - f\right]}$$

$\boxed{\frac{\partial u}{\partial x}}$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left[U_e(x)\, f'\right] = \overbrace{\frac{dU_e(x)}{dx}}^{\frac{\beta}{2-\beta}\,\frac{U_e(x)}{x}}\, f' + U_e(x)\, f'\, \frac{\partial \eta}{\partial x} = \frac{\beta}{2-\beta}\,\frac{U_e(x)}{x}\, f' + U_e(x)\, f'\, \frac{\beta-1}{2-\beta}\,\frac{\eta}{x} =$$

$$= \frac{1}{2-\beta}\,\frac{U_e(x)}{x}\left[\beta f' - (1-\beta)\,\eta f''\right] \longrightarrow \boxed{\frac{\partial u}{\partial x} = \frac{1}{2-\beta}\,\frac{U_e(x)}{x}\left[\beta f' - (1-\beta)\,\eta f''\right]}$$

$$\boxed{\frac{\partial^2 u}{\partial y^2}}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left[U_e(x)\, f''\sqrt{\frac{U_e(x)}{(2-\beta)\,\nu\, x}}\,\right] = U_e(x)\sqrt{\frac{U_e(x)}{(2-\beta)\,\nu\, x}}\, f'''\,\frac{\partial \eta}{\partial y} =$$

$$= U_e(x)\sqrt{\frac{U_e(x)}{(2-\beta)\,\nu\, x}}\, f'''\sqrt{\frac{U_e(x)}{(2-\beta)\,\nu\, x}} = \frac{U_e^2(x)}{(2-\beta)\,\nu\, x}\, f''' \longrightarrow \boxed{\frac{\partial^2 u}{\partial y^2} = \frac{U_e^2(x)}{(2-\beta)\,\nu\, x}\, f'''}$$

Sustituyendo en la $ECdM_x$:

$$\frac{U_e(x)\, f'}{2-\beta}\,\frac{U_e(x)}{x}\left[\beta f' - (1-\beta)\,\eta f''\right] + \sqrt{\frac{\nu\, U_e(x)}{(2-\beta)\, x}}\left[(1-\beta)\,\eta f' - f\,\right] U_e(x)\sqrt{\frac{U_e(x)}{(2-\beta)\,\nu\, x}}\, f'' =$$

$$= \frac{\beta}{2-\beta}\,\frac{U_e^2(x)}{x} + \nu\,\frac{U_e^2(x)}{(2-\beta)\,\nu\, x}\, f'''$$

$$\frac{\beta}{2-\beta}\,\frac{U_e^2(x)}{x}(f')^2 - \cancel{\frac{1-\beta}{2-\beta}\,\frac{U_e^2(x)}{x}\,\eta f' f''} + \cancel{\frac{1-\beta}{2-\beta}\,\frac{U_e^2(x)}{x}\,\eta f' f''} - \frac{1}{2-\beta}\,\frac{U_e^2(x)}{x}\, f f'' = \frac{\beta}{2-\beta}\,\frac{U_e^2(x)}{x} + \frac{1}{2-\beta}\,\frac{U_e^2(x)}{x}\, f'''$$

$$\frac{1}{2-\beta}\,\frac{U_e^2(x)}{x}\, f''' + \frac{1}{2-\beta}\,\frac{U_e^2(x)}{x}\, f f'' - \frac{\beta}{2-\beta}\,\frac{U_e^2(x)}{x}(f')^2 + \frac{\beta}{2-\beta}\,\frac{U_e^2(x)}{x} = 0$$

$$\boxed{f''' + f f'' + \beta\left[1 - (f')^2\right] = 0}$$

→ Diferencia con Blasius

Respecto a las condiciones de contorno :

$\eta = 0$, $f = 0$, $f' = 0$

El problema queda :

$$\boxed{\begin{gathered} f''' + f f'' + \beta \left[1 - \left(f' \right)^2 \right] = 0 \\ \eta = 0 : \; f = 0 \; ; \; f' = 0 \\ \eta \to \infty : \; f' \to 1 \end{gathered}}$$

$$\downarrow \quad \text{VALOR DE CONTORNO} \longrightarrow \text{VALOR INICIAL}$$

$$\boxed{\begin{gathered} f''' + f f'' + \beta \left[1 - \left(f' \right)^2 \right] = 0 \\ \eta = 0 : \; f = 0 \; ; \; f' = 0 \\ f'' = f''_0(\beta) \Big/ \lim_{\eta \to \infty} f' = 1 \end{gathered}}$$

Vamos a obtener C_f, δ_1, δ_2 y H_{12} .

Coeficiente de fricción :

$$\tau_p = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} = \mu \, U_e(x) \sqrt{\frac{U_e(x)}{(2-\beta)\,\nu\, x}} \; f''_0 = \rho \, U_e^2(x) \sqrt{\frac{\nu}{(2-\beta)\, U_e(x)\, x}} \; f''_0$$

Espesor de desplazamiento :

$$dy\Big|_{x=cte} = \sqrt{\frac{(2-\beta)\,\nu x}{U_\infty}}\, d\eta$$

$$\delta_1 = \int_0^\infty \Big(1 - \underbrace{\frac{u}{U_e(x)}}_{f'}\Big)\, dy = \sqrt{\frac{(2-\beta)\,\nu x}{U_\infty}} \int_0^\infty \big(1 - f'\big)\, d\eta = \sqrt{\frac{(2-\beta)\,\nu x}{U_\infty}}\, \big(\eta - f\big)_{\eta\to\infty} \longrightarrow$$

$$\longrightarrow \boxed{\frac{\delta_1}{x} = \sqrt{2-\beta}\, \big(\eta - f\big)_{\eta\to\infty}\, Re_x^{-1/2}}$$

Espesor de cantidad de movimiento :

$$\delta_2 = \int_0^\infty \frac{u}{U_e(x)}\Big(1 - \frac{u}{U_e(x)}\Big)\, dy = \sqrt{\frac{(2-\beta)\,\nu x}{U_\infty}} \int_0^\infty f'\big(1 - f'\big)\, d\eta = \sqrt{\frac{(2-\beta)\,\nu x}{U_\infty}} \Big[f\Big|_0^\infty - \underbrace{\int_0^\infty (f')^2\, d\eta}_{?} \Big]$$

$$f''' + f f'' + \beta\big[1 - (f')^2\big] = 0$$

$$\left\{ \begin{array}{ll} u = f' \longrightarrow & du = f''\, d\eta \\ dv = f'\, d\eta \longrightarrow & v = f \end{array} \right\}$$

$$\downarrow$$

$$f f'' = -\beta\big[1 - (f')^2\big] - f'''$$

$$\downarrow$$

$$\int_0^\infty (f')^2\, d\eta = f f'\Big|_0^\infty - \int_0^\infty f f''\, d\eta = f\Big|_{\eta\to\infty} + \beta \int_0^\infty d\eta - \beta \int_0^\infty (f')^2\, d\eta + \int_0^\infty f'''\, d\eta$$

$$(1+\beta)\int_0^\infty (f')^2\, d\eta = \underbrace{f\Big|_{\eta\to\infty} - \int_0^\infty d\eta} + (1+\beta)\int_0^\infty d\eta + \underbrace{\int_0^\infty f'''\, d\eta}$$

Entonces :

$$\delta_2 = \sqrt{\frac{(2-\beta)\,\nu x}{U_\infty}}\left[f\Big|_0^\infty - \int_0^\infty (f')^2\, d\eta \right]$$

$$\delta_2 = \sqrt{\frac{(2-\beta)\,\nu x}{U_\infty}}\left[\underbrace{f\Big|_0^\infty - \int_0^\infty d\eta}_{-(\eta - f)_{\eta\to\infty}} + \frac{(\eta - f)_{\eta\to\infty} + f_0''(\beta)}{1+\beta} \right]$$

$$\delta_2 = \sqrt{\frac{(2-\beta)\,\nu x}{U_\infty}}\left[(\eta - f)_{\eta\to\infty} + \frac{(\eta - f)_{\eta\to\infty} + f_0''(\beta)}{1+\beta} \right]$$

$$\delta_2 = \sqrt{\frac{(2-\beta)\,\nu x}{U_\infty}}\left[\frac{f_0''(\beta)}{1+\beta} + (\eta - f)_{\eta\to\infty}\left(\frac{1}{1+\beta} - 1 \right) \right]$$

$$\delta_2 = \sqrt{\frac{(2-\beta)\,\nu x}{U_\infty}}\left[\frac{f_0''(\beta)}{1+\beta} - \frac{\beta}{1+\beta}(\eta - f)_{\eta\to\infty} \right]$$

$$\boxed{\frac{\delta_2}{x} = \frac{\sqrt{2-\beta}}{1+\beta}\left[f_0''(\beta) - \beta(\eta - f)_{\eta\to\infty} \right] Re_x^{-1/2}}$$

Factor de forma :

$$H_{12} = \frac{\delta_1}{\delta_2} = \frac{\frac{\delta_1}{x}}{\ldots} = \frac{\sqrt{2-\beta}\,(\eta - f)_{\eta\to\infty}\, Re_x^{-1/2}}{\ldots}$$

$$\ldots\, f_0''(\beta) - \beta(\eta - f)_{\eta\to\infty}$$

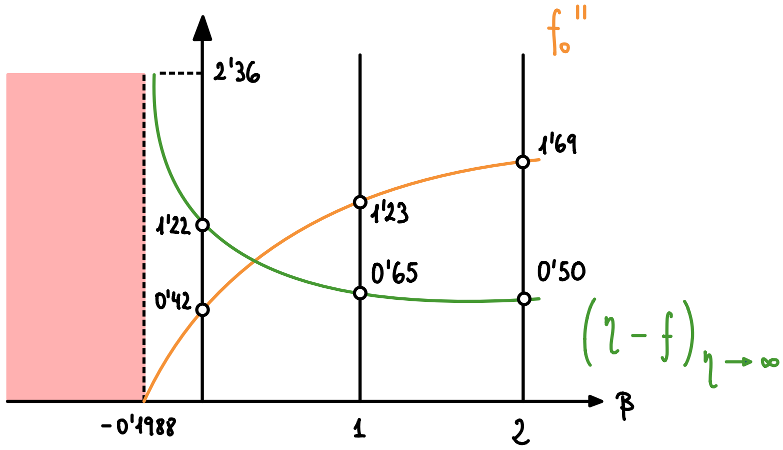
Hay otro parámetro interesante : $\dfrac{\delta_2^2 \dfrac{dU_e}{dx}}{\nu}$

$$\frac{\delta_2^2 \frac{dU_e}{dx}}{\nu} = \frac{x^2}{\nu}\,\frac{2-\beta}{(1+\beta)^2}\left[f_0''(\beta) - \beta(\eta - f)_{\eta\to\infty}\right]^2 \frac{\nu}{U_e(x)\,x}\,\frac{\beta}{2-\beta}\,\frac{U_e(x)}{x}$$

$$\boxed{\frac{\delta_2^2 \frac{dU_e}{dx}}{\nu} = \frac{\beta}{(1+\beta)^2}\left[f_0''(\beta) - \beta(\eta - f)_{\eta\to\infty}\right]^2}$$

$$\frac{\delta_2^2 \frac{dU_e}{dx}}{\nu} \sim \frac{\delta_2^2 \frac{U_e}{x}}{\nu} \sim \underbrace{\left(\frac{\delta_2}{x}\right)^2}_{\sim Re^{-1}} \underbrace{\frac{U_e\, x}{\nu}}_{\sim Re} \sim cte \quad (\text{para cada } \beta)$$

Resumen :



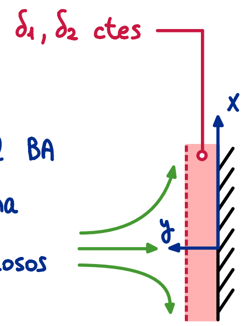
α	β	m	f_0''	$(\eta - f)_{\eta\to\infty}$	$C_f Re_x^{1/2}$	$\frac{\delta_1}{x} Re_x^{1/2}$	$\frac{\delta_2}{x} Re_x^{1/2}$	H_{12}	$\frac{\delta_2^2}{\nu}\frac{dU_e}{dx}$
$\approx 17°$	-0'1988	-0'09	0	2'36	0	3'51	0'87	4'03	-0'068
$0°$	0	0	0'467	1'22	0'66	1'72	0'66	2'59	0

(SEPARACIÓN)

Capa límite cerca de un punto de remanso $\left(\beta = m = 1 \; ; \; U_e = A x \; ; \; \frac{dU_e}{dx} = A\right)$:

$$\frac{\delta_2^2 \frac{dU_e}{dx}}{\nu} = \frac{\delta_2^2 A}{\nu} \sim cte \longrightarrow \begin{cases} \delta_2^2 = 0'85 \dfrac{\nu}{A} = cte \\ \delta_1^2 = H_{12}^2 \delta_2^2 = cte \end{cases}$$

En un ala cerca del BA no hay CL, sino una región de efectos viscosos con δ_1 y δ_2 ctes



Perfil supercrítico de un A320 en aproximación:

$\frac{R_c}{c} \sim 0'10$

$c \sim 4\,m$

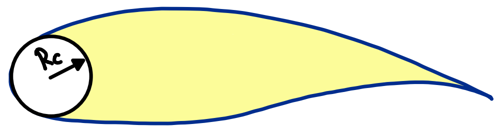
$U_\infty \sim 80\,m/s$

$\nu \sim 1'5 \cdot 10^{-5}\,m^2/s$

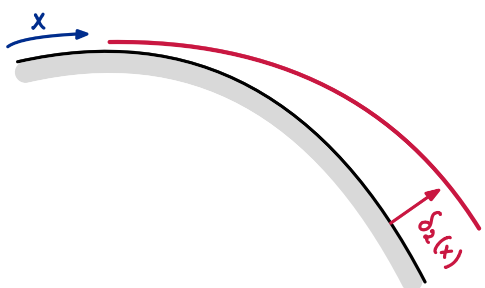
$$U_e = 2 U_\infty \frac{x}{R_c}$$

$$A = \frac{2U_\infty}{R_c} = \frac{20\,U_\infty}{c}$$

$$\left(\frac{\delta_2}{c}\right)^2 \approx \frac{0'085}{20} \frac{\nu}{U_\infty c} \longrightarrow \frac{\delta_2}{c} \sim \frac{1}{2 \cdot 10^4} \longrightarrow \boxed{\delta_2 \sim 0'2\,mm}$$



NOTA



En el caso de $\frac{dU_e}{dx} < 0$ (difusión), si el gradiente adverso se intensifica, entonces $\frac{dU_e}{dx}$ se irá haciendo más negativo $\xrightarrow[\frac{\delta_2^2 \frac{dU_e}{dx}}{\nu} \sim cte]{}$ δ_2 crece $\xrightarrow{\cdots}$ SEPARACIÓN

Suponiendo la placa plana POROSA, la solución de C.L. sigue siendo autosemejante si la velocidad de succión/soplado es de la forma:

$$\frac{v_p}{U_e(x)} = \sqrt{\frac{\nu}{(2-\beta)\, U_e(x)\, x}} \left[(1-\beta)\, \eta f' - f \right] \xrightarrow[\text{semejanza y el escalado}]{\text{Respetando la solución de}} \begin{cases} \dfrac{v_p}{U_e(x)} = \sqrt{\dfrac{\nu}{(2-\beta)\, U_e(x)\, x}}\; \tilde{v}_p \\ \tilde{v}_p = O(1) \end{cases}$$

$$f'' = f_0''(\beta, \tilde{v}_p) \,/\, \lim_{\eta \to \infty} f' = 1$$